Radion effective theory in the detuned Randall-Sundrum model

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Abstract: We compute the two-derivative low-energy effective action for the radion in the (supersymmetric) Randall-Sundrum scenario with detuned brane tensions. At the classical level, a potential automatically stabilizes the distance between the branes. In the supersymmetric case, supersymmetry can be broken spontaneously by a vacuum expectation value for the fifth component of the graviphoton.

Keywords: Supersymmetry Breaking, Supergravity Models, Field Theories in Higher Dimensions.

Contents

1. Introduction

In the original proposal by Randall and Sundrum (RS) [1], five-dimensional anti-de Sitter space (AdS_5) is truncated by two branes located at the fixed points of an $S^1/\mathbb{Z}_2$ orbifold. The branes have opposite tensions, related in magnitude to the bulk cosmological constant. In this scenario, the distance between the branes is a modulus; an additional mechanism is required to stabilize the radius of the circle. In contrast, when the tensions are appropriately detuned, the five-dimensional Einstein's equations automatically determine the radius [2].

In this paper, we derive the low-energy effective theory for the radion in the RS model with detuned brane tensions. We first consider the supersymmetric version of the model, constructed in [3, 4], in which the low-energy dynamics are controlled by an $\mathcal{N} = 1$ supersymmetric effective lagrangian. We compute the Kähler potential and the superpotential for

the radion superfield. We then extend these results to the non-supersymmetric case (where only gravity is present).

This paper is organized as follows. In section 2 we review the supersymmetric RS model with detuned brane tensions, and describe in detail the case where the ground state is AdS_4. In section 3 (and appendix A) we determine the general form of the low-energy effective theory, compatible with the symmetries of the five-dimensional action. The low-energy effective theory turns out to be closely related to no-scale supergravity. In section 4 we use a Kaluza-Klein reduction to compute the low-energy effective action for the bosonic fields, valid to two derivatives. In section 5 we match this result to the supersymmetric effective action and determine all the free parameters. We also show that, in contrast to the tuned case, supersymmetry can be spontaneously broken by a vacuum expectation value (VEV) for the fifth component of the graviphoton. We summarize the results in section 6. In appendix B we discuss corrections to the matching conditions, while in appendix C we present a heuristic derivation of the Kähler potential and superpotential based on the superconformal approach to supergravity.

2. Detuned Randall-Sundrum model

We start by reviewing the supersymmetric RS model with detuned brane tensions [3]. The fifth dimension spans the covering space of an $S^1/\mathbb{Z}_2$ orbifold, parameterized by the coordinate ϕ, with $\phi \in (-\pi, \pi]$. Three-branes, with tensions T_0 and T_π, are placed at the orbifold fixed points, $\phi = 0$ and $\phi = \pi$. The five-dimensional action is given by AdS supergravity, together with brane actions that describe the tensions of the branes. The bosonic part of the action is

$$\begin{aligned} S_{\text{bulk}} &= -M_5^3 \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G} \Big(\frac{1}{2}\mathcal{R} - 6k^2 + \frac{1}{4} F^{MN} F_{MN} + \frac{1}{6\sqrt{6}} \epsilon^{MNPQR} B_M F_{NP} F_{QR} \Big) \\ S_{\text{brane}} &= -T_0 \int d^4x \sqrt{-g_0} - T_\pi \int d^4x \sqrt{-g_\pi}, \end{aligned} \tag{2.1}$$

where g_0 and g_π are the induced metrics on the branes, and B_M is the graviphoton, a $U(1)$ gauge field required by supersymmetry. Supersymmetry restricts the brane tensions to satisfy the bound $|T_{0,\pi}| \leq T$ [3], where T is the "fine-tuned tension," related to the five-dimensional Planck mass by

$$T = 6M_5^3 k. \tag{2.2}$$

When this bound is satisfied, the full bulk-plus-brane theory is invariant under five-dimensional $\mathcal{N} = 2$ supersymmetry in the bulk, restricted to four-dimensional $\mathcal{N} = 1$ supersymmetry on the branes. In this case, the low-energy effective action is $\mathcal{N} = 1$ supersymmetric.

In the original RS model, with tuned brane tensions, the ground-state metric is flat four-dimensional spacetime, warped along the fifth dimension. The Einstein equations are solved for any distance between the branes; the radius is a modulus of the compactification. For detuned brane tensions, four-dimensional flat space is replaced by AdS_4 or dS_4. The AdS_4 ground state arises when $|T_{0,\pi}| < T$; the dS_4 vacuum corresponds to $|T_{0,\pi}| > T$. In the latter

case, we exclude same-sign tensions because they give a warp factor with a zero between the two branes. (We do not consider the other regions for T_0 and T_π because they do not support parallel brane solutions.)

We first consider AdS$_4$. The five-dimensional metric is

$$ds^2 = F(\phi)^2\, g_{mn}\, dx^m dx^n + r_0^2\, d\phi^2, \tag{2.3}$$

where r_0 is the radius of S^1, F is the warp factor, and g_{mn} is the metric of AdS$_4$ with radius L. The warp factor is determined by the five-dimensional Einstein equations,

$$\begin{aligned} FF'' - k^2 r_0^2\, F^2 &= -\frac{2}{T}\Big[kr_0T_0\, F^2\delta(\phi) + kr_0T_\pi\, F^2\delta(\phi-\pi)\Big] \\ F'^2 - k^2 r_0^2\, F^2 &= -\frac{r_0^2}{L^2}. \end{aligned} \tag{2.4}$$

The solution to these equations, defined on the orbifold, is a combination of positive and negative exponentials,

$$F(\phi) = e^{-kr_0|\phi|} + \frac{1}{4k^2L^2}e^{kr_0|\phi|}. \tag{2.5}$$

With our conventions, the radius of the AdS$_4$ metric is related to the tension of the brane at zero by

$$\frac{1}{4k^2L^2} = \frac{T-T_0}{T+T_0}. \tag{2.6}$$

When $T \neq T_0$, the five-dimensional metric is a parametrization of AdS$_5$ that admits a foliation with AdS$_4$ or dS$_4$ slices of varying curvature along the fifth dimension. When $|T_{0,\pi}| < T$, the parameter L is real, the ground state is AdS$_4$, and the five-dimensional theory can be made supersymmetric. In contrast, when $|T_{0,\pi}| > T$, the ground state is dS$_4$. In that case, certain results can be obtained by the analytic continuation $L \to iL$.

An intriguing new feature of the detuned scenario is the fact that the brane tensions fix the radius of the fifth dimension,[1]

$$2\pi kr_0 = \log\frac{(T+T_0)(T+T_\pi)}{(T-T_0)(T-T_\pi)}. \tag{2.7}$$

The tensions can be chosen such that the radius is stabilized in the phenomenologically relevant regime where the warp factor is large. In the tuned case, when $T_0 = -T_\pi = T$, the radius is a modulus, so r_0 is not determined by (2.7). Note that if only one tension is tuned, the critical distance is infinite.

3. Supersymmetric effective action

In this section we present the most general low-energy effective action compatible with the symmetries of the supersymmetric five-dimensional theory. We will see that the $\mathcal{N} = 1$ supersymmetric effective action is determined up to four free parameters.

[1]For simplicity, we restrict the parameters so that $r_0 > 0$. In the AdS$_4$ case, this corresponds to $T_0+T_\pi > 0$; in the dS$_4$ case, to $T_0 + T_\pi < 0$. A similar analysis holds when the signs are reversed.

We start by recalling that G_{mn}, G_{55} and B_5 are even under the orbifold projection, while G_{m5} and B_m are odd. Therefore, at energies below the Kaluza-Klein (KK) scale, the bosonic effective action includes fluctuations of the four-dimensional metric g_{mn}, together with the light modes of G_{55} and B_5. As we will see, the scalar associated with G_{55} is correctly identified as the proper distance between the branes. The other scalar is the zero mode of B_5, since the VEV of B_5 is a modulus of the compactification. The fields G_{m5} and B_m do not give rise to any light modes, but their tadpoles can still contribute to the low-energy effective action.

In the supersymmetric effective theory, the two scalars join with the fifth component of the gravitino to form a chiral supermultiplet, given by

$$D(E,0) \oplus D(E+1,0) \oplus D(E+\frac{1}{2},\frac{1}{2}), \tag{3.1}$$

where $D(E,s)$ denotes the AdS_4 representation labeled by the Casimir E and the spin s. The masses of the particles are related to the eigenvalues E,

$$m_0^2(E) = \frac{E(E-3)}{L^2}, \qquad m_{1/2}(E) = \frac{E-3/2}{L}. \tag{3.2}$$

Since the value of B_5 is a modulus, the corresponding zero mode must be massless. From (3.2), there are only two possible masses for its scalar partner,

$$m_r^2 = -\frac{2}{L^2} \qquad \text{or} \qquad m_r^2 = \frac{4}{L^2}. \tag{3.3}$$

As we will see in the next section, the KK reduction sets $m_r^2 = 4/L^2$. This fixes the mass of the fermionic superpartner to be $m_{1/2} = 2/L$.

The effective action is $\mathcal{N}=1$ supersymmetric, so it is determined by a Kähler potential K and a superpotential P. The bosonic part of the action (setting $M_4 = 1$) takes the form

$$S_{\text{eff}} = -\int d^4x\sqrt{-g}\Big[\frac{1}{2}R + K_{\mathcal{T}\overline{\mathcal{T}}}g^{mn}\partial_m\mathcal{T}\partial_n\overline{\mathcal{T}} + e^K(K^{\mathcal{T}\overline{\mathcal{T}}}D_{\mathcal{T}}PD_{\overline{\mathcal{T}}}\overline{P} - 3P\overline{P})\Big], \tag{3.4}$$

where $\mathcal{T}$ is the lowest component of the radion superfield, and $D_{\mathcal{T}}P = \partial_{\mathcal{T}}P + K_{\mathcal{T}}P$ is the covariant derivative of the superpotential. The imaginary component of $\mathcal{T}$ can be readily identified with the zero mode of B_5. In the next section, we will use the KK reduction to prove that the real part of $\mathcal{T}$ is the proper distance between the branes.

To find K and P, we recall that the *bosonic* part of the five-dimensional action is invariant under the shift $B_5 \to B_5 + \text{constant}$. This follows from the fact that B_5 is derivatively coupled in (2.1), except for an irrelevant $F\tilde{F}$ term. It implies that the Kähler potential is a function of $\mathcal{T}+\overline{\mathcal{T}}$ (up to a Kähler transformation). It also implies that the scalar potential,

$$V(\mathcal{T},\overline{\mathcal{T}}) = e^K(K^{\mathcal{T}\overline{\mathcal{T}}}D_{\mathcal{T}}PD_{\overline{\mathcal{T}}}\overline{P} - 3P\overline{P}), \tag{3.5}$$

cannot depend on the zero mode of B_5. This is an extremely restrictive condition because the superpotential, being a holomorphic function of $\mathcal{T}$, depends explicitly on the zero mode.

In appendix A we show that, in an AdS$_4$ ground state, the most general solution to this condition is[2]

$$\begin{aligned} K(\mathcal{T}+\overline{\mathcal{T}}) &= -3\log[1-c^2\, e^{-a(\mathcal{T}+\overline{\mathcal{T}})}] \\ P(\mathcal{T}) &= p_1 + p_2\, e^{-3a\mathcal{T}}, \end{aligned} \tag{3.6}$$

where $p_1,\, p_2 \in \mathbb{C}$ and $a,\, c \in \mathbb{R}$ are undetermined constants.

With an obvious change of variables, we can cast our result in a more familiar form,

$$\begin{aligned} K(z,\overline{z}) &= -3\log[1-z\overline{z}] \\ P(z) &= p_1' + p_2' z^3. \end{aligned} \tag{3.7}$$

In this parametrization, the Kähler potential is that of no-scale supergravity [5]. In particular, the scalars are the coordinates of the manifold $SU(1,1)/U(1)$. The superpotential is the generalization to AdS space of the superpotential of ordinary no-scale supergravity. The superpotential breaks explicitly the $SU(1,1)$ symmetry of the kinetic term. However, the bosonic action preserves a $U(1)$ subgroup that corresponds to multiplying z by a phase.

4. Bosonic reduction

As we have seen, the low-energy effective theory depends on four constants. To determine the constants, we match the effective action derived from (3.6) to the bosonic reduction of the five-dimensional theory.

4.1 Radion reduction

To compute the low-energy effective action for the radion, we consider the following ansatz for the metric,

$$ds^2 = \Big(F(\phi)^2 + A(x)\Big) g_{mn}(x)\, dx^m dx^n + r_0^2 \left(\frac{F(\phi)^2}{F(\phi)^2 + A(x)}\right)^2 d\phi^2. \tag{4.1}$$

This ansatz is a generalization of the one used in [6]. The field A parameterizes fluctuations in the distance between the branes; the field g_{mn} describes the four-dimensional graviton. For $A=0$, the branes are at the critical distance and the five-dimensional equations of motion require g_{mn} to satisfy the four-dimensional Einstein equations with a cosmological term,

$$R_{mn} - \frac{3}{L^2} g_{mn} = 0. \tag{4.2}$$

The static solution to this equation is AdS$_4$ with radius of curvature L. For $A \neq 0$ there is no static solution, reflecting the fact that a potential is generated for A.

[2] This holds for $m_r^2 = 4/L^2$.

The virtues of this ansatz are three-fold. First, it ensures that there are no tadpoles associated with the G_{m5} components of the metric (see section 4.3). Second, the low-energy effective action is automatically in Einstein frame: to any order in the fields, A does not mix with the four-dimensional graviton. Third, the ansatz respects the boundary conditions that follow from the brane actions,

$$\omega_{ma\hat{5}}\Big|_{\phi\sim 0} = \epsilon(\phi)\frac{T_0}{T}e_{ma}, \qquad \omega_{ma\hat{5}}\Big|_{\phi\sim\pi} = -\epsilon(\phi)\frac{T_\pi}{T}e_{ma}, \tag{4.3}$$

where ω_{MAB} is the spin connection and e_{ma} is the four-dimensional vierbein. As shown in [3], these conditions are also required by local supersymmetry in the five-dimensional theory.

A naive effective action can be found by substituting the ansatz (4.1) into (2.1) and integrating over the fifth dimension. This gives

$$S_4 = -\int d^4x\sqrt{-g}\Big[\frac{M_4^2}{2}R - \frac{3M_4^2}{L^2} + \alpha(A)g^{mn}\partial_m A\partial_n A + (T_0+T_\pi)A^2\Big], \tag{4.4}$$

where

$$\alpha(A) = \frac{T\,r_0}{8\,k}\int_{-\pi}^{\pi}\frac{F(\phi)^2}{\big(F(\phi)^2+A\big)^2}d\phi \tag{4.5}$$

and we have introduced the four-dimensional Planck mass

$$M_4^2 = \frac{T\,r_0}{6\,k}\int_{-\pi}^{\pi}F(\phi)^2 d\phi. \tag{4.6}$$

As advertised, the ansatz gives an effective action in the Einstein frame. Note that the effective action contains a potential for A, enforcing the condition $A=0$ in the ground state.

It is convenient and more physical to write the effective action in terms of the proper distance πr between the branes. The radion field r is related to the fluctuation A as follows,

$$r(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\sqrt{G_{55}}d\phi = \frac{r_0}{2\pi}\int_{-\pi}^{\pi}\frac{F(\phi)^2}{F(\phi)^2+A(x)}d\phi. \tag{4.7}$$

The function α is then simply

$$\alpha(A) = -\frac{\pi}{4}\frac{T}{k}\frac{\partial r}{\partial A}. \tag{4.8}$$

With the change of variables (4.7), the effective action takes the form

$$S_4 = -\int d^4x\sqrt{-g}\Big[\frac{M_4^2}{2}R - \frac{\pi}{4}\frac{T}{k}\Big(\frac{\partial A}{\partial r}\Big)g^{mn}\partial_m r\partial_n r + V(A(r))\Big], \tag{4.9}$$

where

$$V(A(r)) = -\frac{3M_4^2}{L^2} + (T_0+T_\pi)A^2. \tag{4.10}$$

In these expressions, all quantities are expressed as functions of r. The potential has a minimum at $r=r_0$, with four-dimensional cosmological constant

$$\Lambda_4 = V\Big|_{r=r_0} = -\frac{3M_4^2}{L^2}. \tag{4.11}$$

4.2 Graviphoton reduction

The graviphoton reduction proceeds as in [6, 7]. The five-dimensional equations of motion for the graviphoton are

$$\partial_M\big(\sqrt{-G}G^{MN}G^{PQ}F_{NQ}\big)=0, \tag{4.12}$$

where we have dropped the contribution of the Chern-Simons term since it does not contribute to the two-derivative effective action. In the metric background, (4.12) becomes

$$\begin{aligned}&\partial_m\Big(\sqrt{-g}F^{-2}\big(F^2+A\big)^2 g^{mn}F_{5n}\Big)=0\\ &\partial_5\Big(\sqrt{-g}F^{-2}\big(F^2+A\big)^2 g^{mn}F_{5n}\big)+\partial_p\Big(\frac{r_0^2F^2}{F^2+A}\sqrt{-g}g^{pq}g^{mn}F_{qn}\Big)=0,\end{aligned} \tag{4.13}$$

We identify the graviphoton zero mode b with the Aharonov-Bohm phase of B_M around the fifth-dimension

$$b(x)=\frac{1}{\sqrt{6}\pi}\int_{-\pi}^{\pi}B_5 d\phi, \tag{4.14}$$

where the normalization is chosen for later convenience. The field b is gauge invariant for gauge transformations that are periodic on the circle.

Because of the parity assignments, the components B_m do not produce light modes. However, as in [6, 7], they contribute to the effective action through the tadpole $\partial_5 B_m\partial^m B_5$. From equations (4.13), we learn that

$$F_{m5}(x,\phi)=c_m(x)\frac{F(\phi)^2}{\big(F(\phi)^2+A(x)\big)^2} \qquad\qquad F_{mn}=0, \tag{4.15}$$

to lowest order in the derivative expansion. The unknown function c_m can be determined by integrating over the circle. Since all the fields are periodic, we find

$$\partial_m b(x)=\frac{c_m(x)}{\sqrt{6}\pi}\int_{-\pi}^{\pi}\frac{F(\phi)^2}{\big(F(\phi)^2+A(x)\big)^2}d\phi, \tag{4.16}$$

from which it follows that

$$F_{m5}=\sqrt{6}\pi\Big(\int_{\pi}^{\pi}\frac{F(\phi)^2}{\big(F(\phi)^2+A\big)^2}d\phi\Big)^{-1}\frac{F(\phi)^2}{\big(F(\phi)^2+A\big)^2}\ \partial_m b. \tag{4.17}$$

Notice that the constraint $dF=0$ implies that the field strength F_{mn} is already of two-derivative order, so it does not contribute to the two-derivative effective action.

The effective action for the graviphoton is obtained by substituting F_{5m} into (2.1). This gives

$$\begin{aligned}S_4&=-\frac{\pi^2}{2}\frac{T}{kr_0}\int d^4x\sqrt{-g}\Big(\int\frac{F(\phi)^2}{\big(F(\phi)^2+A\big)^2}d\phi\Big)^{-1}g^{mn}\partial_m b\partial_n b\\ &=\frac{\pi}{4}\frac{T}{k}\int d^4x\sqrt{-g}\Big(\frac{\partial A}{\partial r}\Big)g^{mn}\partial_m b\partial_n b.\end{aligned} \tag{4.18}$$

The complete bosonic effective action is obtained by combining (4.9) with (4.18),

$$S_4 = -\int d^4x\sqrt{-g}\Big[\frac{M_4^2}{2}R - \frac{\pi}{4}\frac{T}{k}\Big(\frac{\partial A}{\partial r}\Big)g^{mn}\big(\partial_m r\partial_n r + \partial_m b\partial_n b\big) + V(A(r))\Big]. \tag{4.19}$$

The kinetic terms appear in Kähler form (in fact for any choice of F). This shows that the proper distance r is the correct supersymmetric variable.

4.3 Gravitational tadpoles

At the classical level, the effective action derived above receives corrections associated with integrating out the heavy KK modes. In flat space, the effective action can be organized in a derivative expansion, in which higher-derivative terms are suppressed by appropriate powers of the cut-off. At energies much lower than the cut-off, the higher-derivative terms are negligible. The situation is more subtle in AdS space because a new scale appears, the curvature $1/L^2$. Higher-derivative operators such as R^n do not vanish in the vacuum, but give contributions to the action of order $1/L^{2n}$. Therefore, consistency requires that the two-derivative effective theory also be expanded to first order in $1/L^2$.

To see how this works, let us compute the five-dimensional equations of motion. For simplicity, we work with the variable A. Using the ansatz (4.1), the purely gravitational Einstein equations read

$$\begin{aligned} R_{mn} &= \frac{3}{L^2}g_{mn} - \frac{3}{2}\frac{1}{\big(F(\phi)^2+A\big)^2}\partial_m A\partial_n A - \frac{3}{L^2}\frac{A^2}{F(\phi)^4}g_{mn} \\ \Box A &= \frac{4}{L^2}A + \frac{1}{2}\frac{\partial^m A\,\partial_m A}{F(\phi)^2+A} + \frac{4}{L^2}\frac{A^2}{F(\phi)^2}. \end{aligned} \tag{4.20}$$

(We set $B_M = 0$ because the graviphoton is not relevant for the present discussion.) The $(m5)$ equations are identically zero.

Equations (4.20) show that with our ansatz, the five-dimensional equations of motion are inconsistent. The left-hand side of each equation is a function of the x^m only, while the right-hand side depends on the coordinate ϕ. Note, however, that to *linear* order in A, the five-dimensional equations are consistent; they are satisfied point-by-point in the extra dimension. In particular, the second equation determines the mass of A,

$$m_A^2 = \frac{4}{L^2}. \tag{4.21}$$

This is also the mass of the radion field, since r is related to A by a change of variables.[3] As we have seen, this value of the mass is required by supersymmetry.

The inconsistency of the five-dimensional equations reflects the fact that the Kaluza-Klein gravitons mix with the radion through tadpoles, so the heavy KK fields cannot consistently be set to zero. In fact, eqs. (4.20) are inconsistent at two-field order. This implies that

[3] The same result was previously found in [8]. In fact, our ansatz coincides with theirs to linear order.

the five-dimensional action contains terms with one heavy field coupled to two light fields. Schematically, the action for these fields takes the form,

$$\mathcal{L}_{tadpoles} = A^2 H + (\partial H)^2 + M^2 H^2 + \dots , \tag{4.22}$$

where H stands for a generic heavy field. Note, though, that the $A^2 H$ couplings contain at least two derivatives or two powers of $1/L$. Therefore, integrating out the heavy fields modifies the effective action for the light fields at order ∂^4, ∂^2/L^2, or $1/L^4$. As a consequence, the effective action (4.19) obtained in the previous section should be trusted only up to leading order in ∂^2 and $1/L^2$. To that order, eq. (4.19) becomes

$$S_{\text{eff}} = -\int d^4x \Big[\frac{\hat{M}_4^2}{2} R + 3\pi^2 k^2 \hat{M}_4^2 \, \frac{e^{-\pi k(\mathcal{T}+\overline{\mathcal{T}})}}{(1-e^{-\pi k(\mathcal{T}+\overline{\mathcal{T}})})^2} g^{mn} \partial_m \mathcal{T} \partial_n \overline{\mathcal{T}} + V(\mathcal{T},\overline{\mathcal{T}}) \Big], \tag{4.23}$$

where

$$V(\mathcal{T},\overline{\mathcal{T}}) = -\frac{3\hat{M}_4^2\,(1-e^{-2\pi k r_0})}{L^2} \left[\frac{1-e^{-2\pi k(\mathcal{T}+\overline{\mathcal{T}}-r_0)}}{(1-e^{-\pi k(\mathcal{T}+\overline{\mathcal{T}})})^2} \right], \tag{4.24}$$

$\mathcal{T} = r + ib$ and $\hat{M}_4^2 = M_4^2 = (T/6k^2)(1-e^{-2\pi k r_0})$, to leading order in $1/L^2$.

5. Results

5.1 Supersymmetric action

In section 3 we showed that the most general supersymmetric action, compatible with the symmetries of the five-dimensional theory, is specified by

$$\begin{aligned} K(\mathcal{T}+\overline{\mathcal{T}}) &= -3M_4^2 \log[1-c^2\, e^{-a(\mathcal{T}+\overline{\mathcal{T}})}] \\ P(\mathcal{T}) &= p_1 + p_2\, e^{-3a\mathcal{T}}, \end{aligned} \tag{5.1}$$

where $\mathcal{T} = r + ib$ and we have restored the four-dimensional Planck mass, M_4.

The parameters a, c, p_1 and p_2 must be determined by matching with the KK reduction of the five-dimensional theory. In particular, p_1 and p_2 are set by matching the cosmological constant of the AdS_4 ground state and by requiring that the minimum of the potential be at $r = r_0$. This gives

$$p_1 = \frac{M_4^2\sqrt{1-c^2 e^{-2ar_0}}}{L}, \qquad p_2 = -\frac{c^2 e^{ar_0} M_4^2 \sqrt{1-c^2 e^{-2ar_0}}}{L}\, e^{i\beta}, \tag{5.2}$$

where, for the moment, the phase β is free.

These results fix the bosonic part of the low-energy effective action in terms of a and c. Using (5.1) and (5.2), we find

$$S_{\text{eff}} = -\int d^4x \Big[\frac{M_4^2}{2} R + 3a^2\, c^2 M_4^2 \, \frac{e^{-a(\mathcal{T}+\overline{\mathcal{T}})}}{(1-c^2\, e^{-a(\mathcal{T}+\overline{\mathcal{T}})})^2} g^{mn} \partial_m \mathcal{T} \partial_n \overline{\mathcal{T}} + V(\mathcal{T},\overline{\mathcal{T}}) \Big], \tag{5.3}$$

where the scalar potential is

$$V(\mathcal{T},\overline{\mathcal{T}}) = -\frac{3\,M_4^2\,(1-c^2e^{-2ar_0})}{L^2}\left[\frac{1-c^2\,e^{-2a(\mathcal{T}+\overline{\mathcal{T}}-r_0)}}{(1-c^2\,e^{-a(\mathcal{T}+\overline{\mathcal{T}})})^2}\right]. \tag{5.4}$$

As expected, the potential is independent of b (and the phase β).

The parameters a and c can be found by matching (5.3) with the effective action (4.23). This gives

$$a \;=\; k\pi, \qquad\qquad c \;=\; 1, \tag{5.5}$$

along with $M_4^2 = \hat{M}_4^2$, up to corrections of order $1/L^2$ (see appendix B). The Kähler potential is then[4]

$$K(\mathcal{T}+\overline{\mathcal{T}}) = -3\hat{M}_4^2\log[1-e^{-\pi k(\mathcal{T}+\overline{\mathcal{T}})}], \tag{5.6}$$

while the superpotential is

$$P(\mathcal{T}) = \frac{k\hat{M}_4^3}{L}\sqrt{\frac{6}{T}}\left(1-e^{i\beta}e^{\pi k r_0}e^{-3\pi k\mathcal{T}}\right). \tag{5.7}$$

The superpotential can be understood as the sum of two constant superpotentials, one localized at each of the orbifold fixed points. The radion dependence results from the warping of the metric in the extra dimension. In appendix C we show that this result can be explained in terms of the conformal compensator approach to supergravity.

5.2 Continuation to dS_4

Since it is consistent with the equations of motion to set $B_M = 0$, the effective theory derived above gives the effective action even in the non-supersymmetric case, where the only field is the metric G_{MN}.

The same approach can also be used when the four-dimensional ground state is dS_4 (see also [10]). In this case the absolute values of the brane tensions both exceed the tuned value. As explained in section 2, we take the tensions to have opposite signs. Furthermore, since the five-dimensional theory cannot be made supersymmetric [3], we only consider pure gravity.

To compute the effective theory for the radion, we proceed as in section 4. We use the ansatz (4.1) to derive the effective action, up to two derivatives and leading order in $1/L^2$. From the linearized analysis, we find that the radion has a tachyonic mass [8], $m_R^2 = -4/L^2$, which implies that the dS_4 solution is unstable.

We can now exploit the fact that the five-dimensional equations of motion are identical to the AdS_4 case, with L replaced by iL. Therefore, even though the theory is not supersymmetric, the low-energy effective action can be obtained from the supersymmetric result. The kinetic term remains the same, while the potential changes sign,

$$V(\mathcal{T},\overline{\mathcal{T}}) = \frac{3\hat{M}_4^2(1-e^{-2\pi k r_0})}{L^2}\left[\frac{1-\;e^{-2\pi k(\mathcal{T}+\overline{\mathcal{T}}-r_0)}}{(1-\;e^{-\pi k(\mathcal{T}+\overline{\mathcal{T}})})^2}\right]. \tag{5.8}$$

[4] These results agree with [9], where the authors considered the case $T_0 = -T_\pi$. In this limit, the critical distance between the branes is zero.

Given the Kähler potential (3.6), one can show that no superpotential can give rise to this scalar potential, in accord with the fact that the original theory is not supersymmetric.

5.3 Supersymmetry breaking

The bosonic reduction completely determines the full supersymmetric effective action, up to the single phase β in the superpotential (5.7). Without loss of generality, the value of β can be fixed by demanding that supersymmetry be unbroken when $b = 0$. In [3], it was shown that one can always choose fermionic brane actions in such a way that this is true. Other choices of brane action correspond to shifting the origin of B_5 [11].

In the effective theory, unbroken supersymmetry requires that the covariant derivative of the superpotential vanishes, when evaluated at the minimum of the potential. With the above Kähler potential and superpotential, we find

$$D_{\mathcal{T}} P\Big|_{r=r_0} \sim 1 - e^{i\beta} e^{i3\pi kb}, \tag{5.9}$$

which implies that we must set $\beta = 0$. With $\beta = 0$, supersymmetry is then spontaneously broken when $\langle b \rangle \neq 2n/(3k)$, for n integer. This mechanism of supersymmetry breaking is the generalization to AdS_4 of the no-scale models with constant superpotentials. In the limit where the tensions are tuned, $T_0 = -T_\pi = T$, supersymmetry cannot be broken because the superpotential (5.7) vanishes identically.

In ref. [11], by studying the five-dimensional Killing spinor equations, we found that a non-zero VEV of b breaks supersymmetry. This corresponds to a non-trivial Wilson line for the graviphoton around the fifth dimension. The supersymmetry breaking vanishes in the tuned limit because the Killing spinor equations can always be satisfied in that case. In [11] we also found that b is a periodic variable. The periodicity matches exactly the one derived here.

6. Summary

In this paper we computed the low-energy effective theory for the radion in the supersymmetric Randall-Sundrum scenario with detuned brane tensions, extending the results of [6, 7]. The form of the effective action is determined by $\mathcal{N} = 1$ supersymmetry and by the symmetries of the bosonic five-dimensional theory. The Kähler potential and the superpotential for the radion chiral superfield (to leading order in $1/L^2$) are given by

$$\begin{aligned} K(\mathcal{T}, \overline{\mathcal{T}}) &= -3 \log \left[1 - e^{-\pi k (\mathcal{T} + \overline{\mathcal{T}})} \right] \\ P(\mathcal{T}) &= \frac{k}{L} \sqrt{\frac{6}{T}} \left(1 - e^{\pi k r_0} e^{-3\pi k \mathcal{T}} \right), \end{aligned} \tag{6.1}$$

where $T = r + ib$. The ground state is AdS_4; the potential stabilizes the distance between the branes, while b remains a modulus of the compactification. This supersymmetric model is a

generalization to anti de-Sitter space of the no-scale supergravity models of flat space. Supersymmetry can be broken spontaneously by a VEV for the fifth component of the graviphoton.

The purely gravitational case is obtained setting graviphoton and the fermionic degrees of freedom to zero. The action can also be continued to dS_4, even though the five-dimensional theory is not supersymmetric. The theory is unstable in this case.

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A. AdS no-scale supergravity

In this appendix we derive the Kähler potential and superpotential presented in section 3. The scalar potential for the complex radion field $\mathcal{T}$ is given by[5]

$$V(\mathcal{T},\overline{\mathcal{T}}) = e^K (K^{\mathcal{T}\overline{\mathcal{T}}} D_{\mathcal{T}} P D_{\overline{\mathcal{T}}} \overline{P} - 3P\overline{P}). \tag{A.1}$$

In this expression, the Kähler potential K is a function of $\mathcal{T}+\overline{\mathcal{T}}$, while the superpotential P is an analytic function of $\mathcal{T}$.

In what follows we determine K and P so that V is a function of the real part of $\mathcal{T}$ only. We start by expanding in a power series about the vacuum, which by a shift we take to be at $\mathcal{T}=0$,

$$\begin{aligned} K(\mathcal{T},\overline{\mathcal{T}}) &= \sum_{n=0}^{\infty} k_n \left(\frac{\mathcal{T}+\overline{\mathcal{T}}}{2}\right)^n \\ P(\mathcal{T}) &= \frac{1}{L}\sum_{n=0}^{\infty} p_n \mathcal{T}^n, \end{aligned} \tag{A.2}$$

where $k_n \in \mathbb{R}$ and $p_n \in \mathbb{C}$. We use a residual Kähler transformation to set k_0 and k_1 to zero; we take $k_2 = 2$ to fix the normalization of the kinetic term. We also set $p_1 = 0$ so that supersymmetry is unbroken when $\mathcal{T} = 0$.

We now substitute (A.2) into (A.1) and expand in powers of $x = \mathrm{Re}\,\mathcal{T}$ and $y = \mathrm{Im}\,\mathcal{T}$. To lowest order in the expansion, we must set $p_0 = 1$ to ensure that the ground state corresponds to AdS space of radius L. To next order, we find two possibilities, $p_2 = 0$ and $p_2 = -3/2$. We choose the latter, which gives the correct radion mass, $m_x^2 = 4/L^2$.

[5] In principle, the real part of $\mathcal{T}$ is a generic function of r. The following derivation relies only on the fact that the imaginary part of $\mathcal{T}$ is proportional to the zero mode of B_5.

Further conditions are found by requiring that terms proportional to $x^n y^k$ vanish, for $k \geq 1$. The $x^n y$ terms fix the p_n to be real (up to an irrelevant overall phase). The other terms iteratively determine the coefficients k_n and p_n in terms of k_3 for $n \geq 3$. To see how this works, note that at order n, the parameters (p_n, k_n) appear for the first time. These parameters are fixed by the $x^{n-k}y^k$ terms, for $k > 1$. Explicitly, from the $x^{n-2}y^2$, $x^{n-4}y^4$ terms, we find

$$\begin{aligned} n(n-1)((4n-20)\,p_n - 9k_n) &= g_{n-1}(p_{n-1}, k_{n-1}) \\ n(n-1)(n-2)(n-3)(n-13)\,p_n &= h_{n-1}(p_{n-1}, k_{n-1}), \end{aligned} \tag{A.3}$$

where the functions g_n and h_n are polynomials that depend only on the parameters (p_n, k_n) and lower. (The equations associated with other powers of y are redundant.) The above equations determine p_n and k_n for any n except $n = 13$. For $n = 13$, the equation associated with $x^{n-6}y^6$ supplies the missing relation.

This completes the proof that the Kähler potential and superpotential are determined by two physical parameters, L and k_3 (which is related to the three-point function). By a field redefinition, K and P can be cast in the form presented in section 3,

$$\begin{aligned} K(\mathcal{T} + \overline{\mathcal{T}}) &= -3\log[1 - c^2\, e^{-a(\mathcal{T}+\overline{\mathcal{T}})}] \\ P(\mathcal{T}) &= q_1 + q_2\, e^{-3a\mathcal{T}}, \end{aligned} \tag{A.4}$$

with q_1, $q_2 \in \mathbb{C}$. The six parameters in (A.4) are L and k_3, the scale of $\mathcal{T}$, a shift of x and y, and an overall (irrelevant) phase of P.

B. Corrections to K and P

In this appendix we present evidence that the effective action (4.19) is correct to cubic order in the fluctuations of the radion, and all orders in $1/L^2$. If true, this would allow us to compute the Kähler potential and superpotential to all orders in $1/L^2$. The $1/L^2$ corrections are necessary when treating the higher-derivative terms in the effective action.

We start by considering the action (4.19). The kinetic term for the radion is given by

$$S_{\rm kin} = \frac{\pi}{4}\,\frac{T}{k} \int d^4x \sqrt{-g}\, \Big(\frac{\partial A}{\partial r}\Big) g^{mn} \partial_m r \partial_n r. \tag{B.1}$$

From (4.20), we see that the five-dimensional equations of motion are consistent to linear order in A, and all orders in $1/L^2$. Therefore the heavy-field tadpoles modify the effective action, starting at fourth order in A.

To write the action explicitly, we need to express the derivative $\partial A/\partial r$ as function of r. Therefore we must invert the relation

$$r(A) = \frac{r_0}{2\pi} \int_{-\pi}^{\pi} \frac{F(\phi)^2}{F(\phi)^2 + A} d\phi. \tag{B.2}$$

Since tadpoles affect the action at fourth order, it is sufficient to compute A to quadratic order in r. Expanding (B.2) in powers of A, we find

$$r = r_0\big(1 - I_2 A + I_4 A^2 + \ldots\big), \tag{B.3}$$

where we have introduced the integrals

$$I_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(\phi)^{-n} d\phi. \tag{B.4}$$

Equation (B.3) is readily inverted,

$$A = -\frac{1}{I_2}\left(\frac{r-r_0}{r_0}\right) + \frac{I_4}{I_2^3}\left(\frac{r-r_0}{r_0}\right)^2 + \ldots \tag{B.5}$$

Using this result, we find that the kinetic term (B.1) becomes

$$S_{\rm kin} = -\frac{\pi}{4}\frac{T}{kr_0}\int d^4x\sqrt{-g}\left[\frac{1}{I_2} - \frac{2I_4}{I_2^3}\left(\frac{r-r_0}{r_0}\right) + \ldots\right] g^{mn}\partial_m r\partial_n r. \tag{B.6}$$

The potential term in (4.19) can also be written in terms of r,

$$V(r) = \frac{\pi r_0}{L^2}\frac{T}{k} I_2\, A^2 = \frac{\pi r_0}{L^2}\frac{T}{k}\Big[\frac{(r-r_0)^2}{I_2 r_0^2} - 2\frac{I_4}{I_2^3}\frac{(r-r_0)^3}{r_0^3} + \ldots\Big]. \tag{B.7}$$

The coefficient of the cubic term is precisely the one required by supersymmetry, proving that the effective action obtained by the Kaluza-Klein reduction is supersymmetric to cubic order in fields, to *any* order in $1/L^2$. Moreover, expanding the action (4.19) to fourth order, one can check that it is not supersymmetric to that order.

These results suggest that the effective action for the matter fields is correct to all orders in $1/L^2$ (and cubic order in the fields). Then, by matching with the supersymmetric action, we can compute the corrections to the Kähler potential and the superpotential, to all orders in $1/L^2$. The periodicity of b matches that of the five-dimensional theory [11], so the relation $a = k\pi$ is exact. However, the identification $c = 1$ receives corrections at order $1/L^2$. Matching the cubic action for r with the corresponding terms in the supersymmetric effective theory, we find

$$\frac{1 - c^2\, e^{-2\pi k r_0}}{1 + c^2\, e^{-2\pi k r_0}} = \pi k r_0 \frac{I_2^2}{I_4}. \tag{B.8}$$

Note that in this case, we do not match the Planck mass because it is corrected by the higher-derivative operators proportional to R^n.

C. Conformal compensator approach

The Kähler potential and the superpotential found in this paper have a beautiful explanation in terms of the conformal compensator approach to supergravity (see [7]). In this formalism

[12], one decouples gravity and retains one of the supergravity auxiliary fields. The superspace effective lagrangian for the radion is given by

$$L = \int d^4\theta \overline{\Sigma}\Sigma f(\mathcal{T},\overline{\mathcal{T}}) + \int d^2\theta \Sigma^3 P(\mathcal{T}) + \int d^2\overline{\theta}\, \overline{\Sigma}^3 \overline{P}(\overline{\mathcal{T}}), \tag{C.1}$$

where the chiral superfield Σ is the conformal compensator,

$$\Sigma = 1 + \theta^2 M, \tag{C.2}$$

and we use $\mathcal{T}$ to denote the radion superfield as well as its lowest component. The function $f(\mathcal{T},\overline{\mathcal{T}})$ is related to the Kähler potential by

$$K(\mathcal{T},\overline{\mathcal{T}}) = -3\log\Big[-\frac{f(\mathcal{T},\overline{\mathcal{T}})}{3}\Big]. \tag{C.3}$$

To leading order in $1/L^2$, the warp factor $F(\phi) = e^{-kr_0|\phi|}$. The conformal compensator is then

$$\Sigma = \Sigma_0 e^{-k\mathcal{T}|\phi|}, \tag{C.4}$$

where $\Sigma_0 = 1 + \theta^2 M$. The four-dimensional effective lagrangian is obtained by integrating the conformal compensator over the fifth dimension. The kinetic term is

$$\frac{k}{2}\int_{-\pi}^{\pi} d^4\theta\, d\phi\, \Sigma_0\overline{\Sigma}_0\ (\mathcal{T}+\overline{\mathcal{T}})e^{-k(\mathcal{T}+\overline{\mathcal{T}})|\phi|} = \int d^4\theta\ \Sigma_0\overline{\Sigma}_0\left(1 - e^{-\pi k(\mathcal{T}+\overline{\mathcal{T}})}\right). \tag{C.5}$$

This gives the same Kähler potential as found in the body of this paper. The superpotential can be written as the sum of two brane-localized constants,

$$\int_{-\pi}^{\pi} d^2\theta\, d\phi \Sigma_0^3\, e^{-3k\mathcal{T}|\phi|}\big[p_1\ \delta(\phi) + p_2\ \delta(\phi-\pi)\big] = \int d^2\theta \Sigma_0^3\big[p_1 + p_2\ e^{-3k\pi\mathcal{T}}\big]. \tag{C.6}$$

In each case, the correct radion dependence is given by the conformal compensator.

References

[1] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83**, 3370 (1999).

[2] N. Kaloper, "Bent domain walls as brane-worlds," Phys. Rev. D **60**, 123506 (1999).

[3] J. Bagger and D. V. Belyaev, "Supersymmetric branes with (almost) arbitrary tensions," Phys. Rev. D **67**, 025004 (2003);
J. Bagger and D. V. Belyaev, "Twisting warped supergravity," JHEP **0306**, 013 (2003).

[4] R. Altendorfer, J. Bagger and D. Nemeschansky, "Supersymmetric Randall-Sundrum scenario," Phys. Rev. D **63**, 125025 (2001);
T. Gherghetta and A. Pomarol, "Bulk fields and supersymmetry in a slice of AdS," Nucl. Phys. B **586**, 141 (2000);
A. Falkowski, Z. Lalak and S. Pokorski, "Supersymmetrizing branes with bulk in five-dimensional supergravity," Phys. Lett. B **491**, 172 (2000).

[5] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, "Naturally vanishing cosmological constant in $\mathcal{N} = 1$ supergravity," Phys. Lett. B **133**, 61 (1983);
J. Ellis, A. B. Lahanas, D. V. Nanopoulos and K. Tamvakis, "No-scale supersymmetric Standard Model," Phys. Lett. B **134**, 429 (1984);
J. R. Ellis, C. Kounnas, D. V. Nanopoulos, "Phenomenological $SU(1,1)$ supergravity," Nucl. Phys. B **241**, 406 (1984).

[6] J. Bagger, D. Nemeschansky, R. J. Zhang, "Supersymmetric radion in the Randall-Sundrum scenario," JHEP **0108**, 057 (2001).

[7] M. A. Luty and R. Sundrum, "Hierarchy stabilization in warped supersymmetry," Phys. Rev. D **64**, 065012 (2001).

[8] Z. Chacko, P. J. Fox, "Wave function of the radion in dS and AdS braneworlds," Phys. Rev. D **64**, 024015 (2001).

[9] P. Brax, N. Chatillon, "Detuned branes and supersymmetry breaking," [hep-th/0309117].

[10] M. Ito, "Scalar potential from de sitter brane in 5D and effective cosmological constant," [hep-th/0311006].

[11] J. Bagger and M. Redi, "Supersymmetry breaking by Wilson lines in AdS_5," to appear in Phys. Lett. B, [hep-th/0310086].

[12] S. Ferrara and P. van Nieuwenhuizen, "Tensor calculus for supergravity," Phys. Lett. B **76**, 404 (1978);
K.S. Stelle and P.C. West, "Relation between vector and scalar multiplets and gauge invariance in supergravity," Nucl. Phys. B **145**, 175 (1978);
E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, "Yang-Mills theories with local supersymmetry: lagrangian, transformation laws and superhiggs effect," Nucl. Phys. B **212**, 413 (1983).